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An Improved Receding Horizon Control for Time-Delay Systems

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Abstract In this paper, we propose an improved version of receding horizon control for systems with state delays. The proposed control guarantees closed-loop stability for a wider class of state-delay systems than the existing one. For expanded applications, a more generalized cost function, with three terminal weighting terms, is employed and minimized. Terminal weighting matrices are chosen to achieve the property that the optimal cost monotonically decreases with time. It turns out that the stability condition depends on the delay size and then it is less conservative than the existing delay-independent one. The simulation study shows that the proposed control scheme guarantees closed-loop stability even for state-delay systems that cannot be stabilized by the existing receding horizon control.

Keywords Calculus of variations and optimal control · Receding horizon control (RHC) · Delay-dependent controls · Linear matrix inequality (LMI) · State-delay systems · Terminal weighting matrices

Mathematics Subject Classification 49J15 · 49N05

1 Introduction

For delay-free systems, the receding horizon control (RHC) has received considerable attention [1–4] because of its many advantages, including ease of computation, good

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tracking performance, and I/O constraint handling capability, compared with the popular infinite horizon optimal controls. Since reference signals are only available over the finite future time horizon and a variety of constraints should be imposed for practical applications, the RHC based on the finite horizon optimal controls is preferable to the infinite horizon ones. Therefore, the RHC has been widely used, particularly in the chemical process industries [1].

There are several previous results on the RHC for systems with state delays. A simple control method based on the receding horizon concept appears in [5]. However, it does not have a state weighting term in the cost function. Furthermore, it does not guarantee closed-loop stability by design, and hence stability can be checked only after the controller is designed. For guaranteed stability by design, the RHC with both state and input weighting terms in the cost function is proposed in [6]. This approach is also extended to the receding horizon H_∞ control in [7]. For input-delay systems, a primitive version appears in [8] and is generalized in [9] to enlarge the feasibility set.

In RHC, terminal weighting matrices in the cost function are crucial for stability. The cost functions employed in [6–10] include two terminal weighting matrices and the LMI conditions therein for obtaining stability-guaranteeing terminal weighting matrices are delay independent and thus conservative since information on the delay size is not utilized. In other words, closed-loop stability is only guaranteed for a limited class of delay systems. This implies that a certain class of state-delay systems cannot be stabilized. It would be meaningful to make use of the delay size for reduction of such conservatism. The cost function employed in this paper has three terminal weighting matrices in order to enlarge the class of state-delay systems that can be handled through the RHC.

This paper is organized as follows: In Sect. 2, the solution to the proposed RHC is obtained through the calculus of variations. In Sect. 3, a linear matrix inequality (LMI) condition is proposed to obtain stability-guaranteeing terminal weighting matrices. In Sect. 4, the stability of the proposed RHC is investigated. In Sect. 5, a numerical example is provided to illustrate the reduced conservatism of the proposed RHC. Finally, we draw conclusions in Sect. 6.

2 Receding Horizon Control for State-Delay Systems

Consider a time-invariant and continuous time-delay system without input and state constraints

$$\dot{x}(t) = Ax(t) + A_1 x(t-h) + Bu(t), \quad x(\tau) = \phi(\tau), \ \tau \in [-h, 0], \tag{1}$$

where $\phi(t)$ is a differentiable function, and $h > 0$, $x(t) \in \mathbb{R}^n$, and $u(t) \in \mathbb{R}^m$ are the delay size, the state, and the input, respectively. In order to obtain an RHC, we first consider a finite horizon cost function represented by

$$J(x_{t_0},u,t_0,t_f)=\int_{t_0}^{t_f}[x^T(\tau)Qx(\tau)+u^T(\tau)Ru(\tau)]d\tau+x^T(t_f)F_1x(t_f)$$
$$+\int_{t_f-h}^{t_f}x^T(\alpha)F_2x(\alpha)d\alpha+\int_{-h}^{0}\int_{t_f+\beta}^{t_f}\dot{x}^T(\alpha)F_3\dot{x}(\alpha)d\alpha d\beta, \quad (2)$$

where $x_{t_0}=x(t_0+s)$, $s\in[-h,0]$, is an initial function, t_0 is the initial time, t_f is the terminal time, and $Q>0$, $R>0$, $F_1>0$, $F_2>0$, and $F_3>0$. $t_f-t_0>0$ is called the horizon size. It is noted that the cost function (2) has three terminal weighting terms. The optimal control minimizing the cost function (2) and the corresponding optimal cost will be denoted by $u^*(\tau)$, $t_0\le\tau\le t_f$, and $J^*(x_{t_0},t_0,t_f)$, respectively. It is apparent that $J^*(x_{t_0},t_0,t_f)=J(x_{t_0},u^*,t_0,t_f)$. The RHC is then obtained by minimizing the cost function (2) with the initial time t_0 and the terminal time t_f replaced by the current time t and $t+T_p$, respectively, where T_p is a (prediction) horizon size. As will be shown later, the stability of the proposed RHC depends on the choice of three terminal weighting matrices F_1, F_2, and F_3.

As a special case, the RHC minimizing the following cost function without the third weighting term in (2) was proposed in [6]. This corresponds to (2) with $F_3=0$. However, only a limited class of state-delay systems satisfies the LMI condition on the stability-guaranteeing terminal weighting matrices provided in [6] with $F_3=0$. This comes from the fact that the condition is delay-independent and thus too strong. As will be seen in the next section, we can derive a delay-dependent condition on stability-guaranteeing terminal weighting matrices by introducing the third weighting terms parameterized by F_3. This, in turn, enables the proposed delay-dependent RHC to stabilize a wider class of state-delay systems than the existing delay-independent one in [6].

First, the solution to the finite horizon optimal control problem will be derived through the calculus of variations. Second, RHC will be obtained from the derived solution. Throughout this paper, we assume $t_f-t_0\le h$ for simple computation and less memory size in computing. Since t_f-t_0 is replaced with the prediction horizon size T_p in the RHC, T_p is adjusted to be less than or equal to h. After some tedious algebraic computations, it can be easily seen that instead of (2), we have only to minimize a new cost function $\bar{J}(x_{t_0},u,t_0,t_f)$ represented by

$$\int_{t_0}^{t_f}[x^T(\tau)\bar{Q}(\tau)x(\tau)+2x^T(\tau)N(\tau)u(\tau)+u^T(\tau)\bar{R}(\tau)u(\tau)$$
$$+2x^T(\tau)U(\tau)x(\tau-h)+2u^T(\tau)V(\tau)x(\tau-h)]dt+x^T(t_f)F_1x(t_f), \quad (3)$$

where $\bar{Q}(\tau)$, $\bar{R}(\tau)$, $N(\tau)$, $U(\tau)$, and $V(\tau)$ are defined as

$$\bar{Q}(\tau):=Q+F_2+f(\tau)A^TF_3A,\ \bar{R}(\tau):=R+f(\tau)B^TF_3B,\ N(\tau):=f(\tau)A^TF_3B, \quad (4)$$
$$U(\tau):=f(\tau)A^TF_3A_1,\quad V(\tau):=f(\tau)B^TF_3A_1,\quad f(\tau):=(\tau-t_f+h). \quad (5)$$

It is apparent that the optimal control minimizing the cost function $\bar{J}(x_{t_0}, u, t_0, t_f)$ also minimizes the original one $J(x_{t_0}, u, t_0, t_f)$.

From now on, we will take the existing variational approach of optimal control theory [11] to obtain the optimal control for the state-delay system (1). The necessary and sufficient conditions for optimality are given as follows:

$$\dot{x}(\tau) = Ax(\tau) + A_1 x(\tau - h) + Bu(\tau), \tag{6}$$

$$\dot{p}(\tau) = -\bar{Q}(\tau)x(\tau) - N(\tau)u(\tau) - U(\tau)x(\tau - h) - A^T p(\tau), \tag{7}$$

$$0 = N^T(\tau)x(\tau) + \bar{R}(\tau)u(\tau) + V(\tau)x(\tau - h) + B^T p(\tau), \quad p(t_f) = F_1 x(t_f), \tag{8}$$

for $t_0 \le \tau \le t_f$, where $p(t)$ is the costate variable. From (8), the optimal control is represented by

$$u(\tau) = -\bar{R}^{-1}(\tau)\Big[N^T(\tau)x(\tau) + V(\tau)x(\tau - h) + B^T p(\tau)\Big]. \tag{9}$$

Substituting (9) into (6) and (7) yields the following set of $2n$ linear differential equations:

$$\begin{bmatrix} \dot{x}(\tau) \\ \dot{p}(\tau) \end{bmatrix} = \begin{bmatrix} A - B\bar{R}^{-1}(\tau)N^T(\tau) & -B\bar{R}^{-1}(\tau)B^T \\ -\bar{Q}(\tau) + N(\tau)\bar{R}^{-1}(\tau)N^T(\tau) & -A^T + N(\tau)\bar{R}^{-1}(\tau)B^T \end{bmatrix} \times \begin{bmatrix} x(\tau) \\ p(\tau) \end{bmatrix} + \begin{bmatrix} A_1 - B\bar{R}^{-1}(\tau)V(\tau) \\ N(\tau)\bar{R}^{-1}(\tau)V(\tau) - U(\tau) \end{bmatrix} x(\tau - h), \tag{10}$$

where $\times$ denotes the multiplication. In order to construct a batch form from the recursive one (10), let us define the following matrix functions:

$$\mathbf{H}(\tau) := \begin{bmatrix} A - B\bar{R}^{-1}(\tau)N^T(\tau) & -B\bar{R}^{-1}(\tau)B^T \\ -\bar{Q}(\tau) + N(\tau)\bar{R}^{-1}(\tau)N^T(\tau) & -A^T + N(\tau)\bar{R}^{-1}(\tau)B^T \end{bmatrix},$$

$$\bar{A}_1(\tau) := A_1 - B\bar{R}^{-1}(\tau)V(\tau), \quad \bar{A}_2(\tau) := N^T(\tau)\bar{R}^{-1}(\tau)V(\tau) - U(\tau),$$

and the state transition matrix $\Phi(\cdot, \cdot)$ of the system as follows:

$$\begin{bmatrix} \dot{x}(\tau) \\ \dot{p}(\tau) \end{bmatrix} = \mathbf{H}(\tau) \begin{bmatrix} x(\tau) \\ p(\tau) \end{bmatrix}.$$

replacing τ and t_0 with t_f and τ, respectively, using the boundary condition, and then solving for $p(\tau)$ yield

$$p(\tau) = W_1(\tau)x(\tau) + \int_{-h}^{t_f - \tau - h} W_2(\tau, s)x(\tau + s)ds, \tag{11}$$

where $W_1(\tau)$ and $W_2(\tau, s)$ will be determined later on. After substitution of (11) into (9), the optimal control $u^*(\tau)$ for $t_0 \le \tau \le t_f$ can be obtained. Then, we are in a position to obtain the RHC. If the initial time t_0 and the terminal time t_f are replaced with t and $t+T_p$, respectively, and a running variable τ is fixed to t, we can obtain the following RHC:

$$\begin{aligned} u^*(t) = -[R+(h-T_p)B^T F_3 B]^{-1} B^T \bigg\{ &[W_1(t)+(h-T_p)F_3 A]x(t) \\ &+ \int_{-h}^{T_p-h} W_2(t,s)x(t+s)ds + (h-T_p)F_3 A_1 x(t-h)\bigg\}, \end{aligned} \tag{12}$$

where $W_1(t)$ and $W_2(t,s)$ are also modified just by replacing the initial and terminal time variables. From (4) to (5), we can show that $\Phi(t+T_p,t)$ is a constant matrix, which implies that it is independent of t. Therefore, we can denote $W_1(t)$ by W_1. Similarly, we can show that $W_2(t,s)$ is also independent of t. Therefore, we denote $W_2(t,s)$ by $W_2(s)$. Finally, we obtain

$$\begin{aligned} u^*(t) = -[R+(h-T_p)B^T F_3 B]^{-1} B^T \bigg\{ &[W_1+(h-T_p)F_3 A]x(t) \\ &+ \int_{-h}^{T_p-h} W_2(s)x(t+s)ds + (h-T_p)F_3 A_1 x(t-h)\bigg\}. \end{aligned} \tag{13}$$

In order to compute $W_1(\tau)$ and $W_2(\tau,s)$ in (11), we have to determine the state transition matrix for $\mathbf{H}(\tau)$, which may not be numerically stable because of the form of $\mathbf{H}(\tau)$. In the following theorem, we provide an alternative method to obtain $W_1(\tau)$ and $W_2(\tau,s)$ in terms of nonlinear matrix partial differential equations solved backward in time.

Theorem 2.1 *The matrix $W_1(\tau)$ and $W_2(\tau,s)$ satisfy the partial differential equations*

$$\begin{aligned} &\dot W_1(\tau) - [W_1(\tau)+f(\tau)A^T F_3]B\bar R^{-1}(\tau)B^T[W_1(\tau)+f(\tau)F_3 A] \\ &\quad + A^T W_1(\tau) + W_1(\tau)A + \bar Q(\tau) = 0 \end{aligned} \tag{14}$$

$$\begin{aligned} &\left(\frac{\partial}{\partial \tau} - \frac{\partial}{\partial s}\right) W_2(\tau,s) + A^T W_2(\tau,s) \\ &\quad - [W_1(\tau)+f(\tau)F_3 A]^T B\bar R^{-1}(\tau)B^T W_2(\tau,s) = 0, \end{aligned} \tag{15}$$

with boundary conditions

$$\begin{aligned} W_2(\tau,-h) = {} & W_1(\tau)A_1 + f(\tau)A^T F_3 A_1 \\ & - f(\tau)[W_1(\tau)+f(\tau)F_3 A]^T B\bar R(\tau)^{-1}B^T F_3 A_1, \quad W_1(t_f) = F_1, \end{aligned} \tag{16}$$

where $t_0 \le \tau \le t_f$ and $-h \le s \le t_f - \tau - h$.

Proof Taking the derivative of (11)and substituting (11) into (7), we can easily see that $W_1(\tau)$ and $W_2(\tau,s)$ should satisfy (14) and (15) with boundary condition (16). The boundary condition (16) follows from the relationships (8) and (11). This completes the proof. □

3 Monotonicity Condition of the Optimal Cost

In this section, we present a condition on the terminal weighting matrices F_1, F_2, and F_3, under which the optimal cost $J^*(x_{t_0}, t_0, \sigma)$ decreases as the terminal time σ increases. We will call it a monotonicity condition of the optimal cost on the terminal weighting matrices.

Theorem 3.1 *Assume that F_1, F_2, and F_3 in (2) satisfy the following matrix inequality for some K, K_1, T_1, T_2, Y_1, Y_2, Y_3:*

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & hY_1 \\ \star & \Phi_{22} & \Phi_{23} & hY_2 \\ \star & \star & \Phi_{33} & hY_3 \\ \star & \star & \star & -hF_3 \end{bmatrix} < 0, \tag{17}$$

where $\star$ denotes the entries of a symmetric matrix and

$$\begin{aligned} \Phi_{11} &:= F_2 + Q + K^T RK + Y_1 + Y_1^T - T_1(A + BK) - (A + BK)^T T_1^T \\ \Phi_{12} &:= F_1 + Y_2^T + T_1 - (A + BK)^T T_2^T \\ \Phi_{13} &:= Y_3^T - Y_1 - T_1(A_1 + BK_1) + K^T RK_1, \quad \Phi_{22} := hF_3 + T_2 + T_2^T, \\ \Phi_{23} &:= -Y_2 - T_2(A_1 + BK_1), \quad \Phi_{33} := -F_2 - Y_3 - Y_3^T + K_1^T RK_1. \end{aligned}$$

The optimal cost $J^*(x_{t_0}, t_0, \sigma)$ satisfies the following monotonicity property:

$$\frac{\partial J^*(x_{t_0}, t_0, \sigma)}{\partial \sigma} < 0, \quad \sigma > t_0. \tag{18}$$

Proof In (18) the derivative is computed as follows:

$$\begin{aligned} \frac{\partial J^*(x_{t_0}, t_0, \sigma)}{\partial \sigma} &= \lim_{\Delta \to 0} \frac{1}{\Delta} \bigg\{ \int_{t_0}^{\sigma+\Delta} \Big[\bar{x}^T(\tau) Q \bar{x}(\tau) + \bar{u}^T(\tau) R \bar{u}(\tau) \Big] d\tau \\ &\quad + \bar{x}^T(\sigma+\Delta) F_1 \bar{x}(\sigma+\Delta) + \int_{\sigma+\Delta-h}^{\sigma+\Delta} \bar{x}^T(\alpha) F_2 \bar{x}(\alpha) d\alpha \\ &\quad + \int_{-h}^{0} \int_{\sigma+\Delta+\beta}^{\sigma+\Delta} \dot{\bar{x}}^T(\alpha) F_3 \dot{\bar{x}}(\alpha) d\alpha d\beta \\ &\quad - \int_{t_0}^{\sigma} \Big[\hat{x}^T(\tau) Q \hat{x}(\tau) + \hat{u}^T(\tau) R \hat{u}(\tau) \Big] d\tau - \hat{x}^T(\sigma) F_1 \hat{x}(\sigma) \\ &\quad - \int_{\sigma-h}^{\sigma} \hat{x}^T(\alpha) F_2 \hat{x}(\alpha) d\alpha - \int_{-h}^{0} \int_{\sigma+\beta}^{\sigma} \dot{\hat{x}}^T(\alpha) F_3 \dot{\hat{x}}(\alpha) d\alpha d\beta \bigg\}, \end{aligned}$$

where $\bar{u}(\cdot)$ and $\hat{u}(\cdot)$ are the optimal controls, which minimize the cost function for the terminal time $\sigma + \Delta$ and σ, respectively. $\bar{x}(\cdot)$ and $\hat{x}(\cdot)$ are state trajectories generated when the system is controlled by $\bar{u}(\cdot)$ and $\hat{u}(\cdot)$, respectively. Assume that the control $\bar{u}(\tau)$ is replaced by $\tilde{u}(\tau)$, which is given by

$$\tilde{u}(\tau) = \begin{cases} \hat{u}(\tau) & , \quad t_0 \le \tau < \sigma \\ K\hat{x}(\tau) + K_1\hat{x}(\tau - h) & , \quad \sigma \le \tau \le \sigma + \Delta \end{cases} . \tag{19}$$

Note that $\hat{x}(\tau)$ for $\tau \in [\sigma, \sigma + \Delta]$ is the state trajectory resulting from $\tilde{u}(\tau)$. $\tilde{u}(\tau)$ is not an optimal control in case that the terminal time is $\sigma + \Delta$. Therefore, we have

$$\begin{aligned} \frac{\partial J^*(x_{t_0}, t_0, \sigma)}{\partial \sigma} \le\ & \hat{x}^T(\sigma)Q\hat{x}(\sigma) + 2\dot{\hat{x}}^T(\sigma)F_1\hat{x}(\sigma) \\ & + [K\hat{x}(\sigma) + K_1\hat{x}(\sigma - h)]^T R[K\hat{x}(\sigma) + K_1\hat{x}(\sigma - h)] + \hat{x}^T(\sigma)F_2\hat{x}(\sigma) \\ & - \hat{x}^T(\sigma - h)F_2\hat{x}(\sigma - h) + h\dot{\hat{x}}^T(\sigma)F_3\dot{\hat{x}}(\sigma) - \int_{\sigma-h}^{\sigma} \dot{\hat{x}}^T(\alpha)F_3\dot{\hat{x}}(\alpha)d\alpha, \end{aligned}$$

where the upper bound can be easily converted to the LMI (17). This completes the proof. □

In the following theorem, we convert the matrix inequality (17) into an equivalent form, which is easier to solve.

Theorem 3.2 *Assume that there exist* $L_1 > 0$, L_2, L_3, N_1, N_2, N_3, V, V_1, U, *and* > 0 *such that*

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & hN_1 & L_1Q^{\frac{1}{2}} & V^TR^{\frac{1}{2}} & hL_2^T \\ \star & \Gamma_{22} & \Gamma_{23} & hN_2 & 0 & 0 & hL_3^T \\ \star & \star & \Gamma_{33} & hN_3 & 0 & V_1^TR^{\frac{1}{2}} & 0 \\ \star & \star & \star & \Gamma_{44} & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & \star & -hU \end{bmatrix} < 0, \tag{20}$$

where Γ_{11}, Γ_{12}, Γ_{13}, Γ_{22}, Γ_{23}, Γ_{33}, *and* Γ_{44} *are given by*

$$\begin{aligned} &\Gamma_{11} = L_2 + L_2^T + N_1 + N_1^T + W, \quad \Gamma_{12} = L_3 + L_2^T - (AL_1 + BV)^T + N_2^T, \\ &\Gamma_{13} = -N_1 + N_3^T, \quad \Gamma_{22} = L_3 + L_3^T, \quad \Gamma_{23} = -(A_1L_1 + BV_1) - N_2, \\ &\Gamma_{33} = -W - N_3 - N_3^T, \quad \Gamma_{44} = -hL_1U^{-1}L_1. \end{aligned}$$

Then the matrix inequality (17) *of Theorem* 3.1 *is feasible and terminal weighting matrices satisfying the inequality* (17) *can be chosen to be* $F_1 = L_1^{-1}$, $F_2 = L_1^{-1}WL_1^{-1}$, *and* $F_3 = U^{-1}$.

Proof The feasibility of (17) requires $T_2 + T_2^T < 0$, which in turn requires T_2 to be nonsingular. Define

$$\begin{bmatrix} F_1 & 0 \\ T_1^T & T_2^T \end{bmatrix}^{-1} = L = \begin{bmatrix} L_1 & 0 \\ L_2 & L_3 \end{bmatrix}.$$

It is noted that L is nonsingular if (17) is feasible. Let's pre- and post-multiply (17) by $\mathrm{diag}\{L^T, L_1, L_1\}$ and $\mathrm{diag}\{L, L_1, L_1\}$, respectively, and introduce some change of variables such that

$$L_1 := F_1^{-1}, \quad W := L_1 F_2 L_1, \quad U := F_3^{-1}, \quad V := K L_1, \quad V_1 := K_1 L_1,$$
$$L^T \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} L_1 := \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}, \quad L_1 Y_3 L_1 := N_3.$$

The matrix inequality (17) is then equivalently changed to (20) through the Schur complement. This implies that (17) is feasible if (20) is feasible. This completes the proof. □

4 Stability of RHC

In this section, we investigate the stability of the RHC (13) using the result of the previous section.

Theorem 4.1 *Given $Q > 0$ and $R > 0$, if $\frac{\partial J^*(x_{t_0}, t_0, \sigma)}{\partial \sigma} \le 0$ for $\sigma > t_0$, the system (1) controlled by the RHC (13) is asymptotically stable.*

Proof If $\frac{\partial J^*(x_{t_0}, t_0, \sigma)}{\partial \sigma} \le 0$ for $\sigma > t_0$, we have

$$J^*(x_t, t, t+T_p) \ge \int_t^{t+\theta} [x^T(\tau) Q x(\tau) + u^{*T}(\tau) R u^*(\tau)] d\tau + J^*(x_{t+\theta}, t + \theta, t + T_p + \theta).$$

Rearranging the above inequality, dividing it by θ, and making x go to ∞, we obtain $dJ^*(x_t, t, t+T_p)/dt \le -[x^T(t) Q x(t) + u^{*T}(t) R u^*(t)]$, which shows that $J^*(x_t, t, t+T_p)$ is non-increasing. Because $J^*(x_t, t, t+T_p) \ge 0$, $J^*(x_t, t, t+T_p) \to c$ and $dJ^*(x_t, t, t+T_p)/dt \to 0$ as $t \to \infty$, where c is a non-negative constant. From this, it is clear that $x(t) \to 0$ and $u^*(t) \to 0$ as $t \to \infty$. Therefore, the closed-loop system is asymptotically stable. This completes the proof. □

From the sufficient conditions for the monotonic cost, the following theorem is obtained.

Theorem 4.2 *If the matrix inequality (20) is feasible for given $Q > 0$ and $R > 0$, then the proposed RHC in (13) designed with terminal weighting matrices $F_1 = L_1^{-1}$, $F_2 = L_1^{-1} W L_1^{-1}$, and $F_3 = U^{-1}$ asymptotically stabilizes the system (1).*

Proof From Theorem 4.1, we see that $\frac{\partial J^*(x_{t_0},t_0,\sigma)}{\partial\sigma} \leq 0$ for $\sigma > t_0$ is sufficient for asymptotic stability of the system controlled by the RHC. Feasibility of (20) is equivalent to that of (17). Therefore, if we choose terminal weighting matrices such that $F_1 = L_1^{-1}$, $F_2 = L_1^{-1} W L_1^{-1}$, $F_3 = U^{-1}$, the optimal cost $J^*(x_{t_0}, t_0, \sigma)$ satisfies (18) and resultant RHC asymptotically stabilizes the system. This completes the proof. □

5 A Numerical Example

In this section, we provide a numerical example to show the features of the proposed RHC.

Let us consider a liquid monopropellant rocket motor with a pressure feeding system. A linearized version of the feeding system and combustion chamber equations, assuming nonsteady flow, is given as follows [12]:

$$\dot{\varphi}(t) = (\gamma - 1)\varphi(t) - \gamma\varphi(t-h) + \mu(t-h), \quad \dot{\mu}_1(t) = \frac{1}{\xi J}\left[-\psi(t) + \frac{p_0 - p_1}{2\Delta p}\right],$$
$$\dot{\mu}(t) = \frac{1}{(1-\xi)J}[-\mu(t) + \psi(t) - P\varphi(t)], \quad \dot{\psi}(t) = \frac{1}{E}[\mu_1(t) - \mu(t)], \tag{21}$$

where θ_g is the gas residence time in the steady operation, $h = \bar{\tau}/\theta_g$ is the reduced time lag with $\bar{\tau}$ the value of the time lag in steady operations, $\varphi(t) = [p(t) - \bar{p}]/\bar{p}$, with $p(t)$ the instantaneous pressure in the combustion chamber and $\bar{p}$ the pressure in the combustion chamber in steady operation, $\mu(t) = (\dot{m}_i - \bar{\dot{m}})/\bar{\dot{m}}$, with $\dot{m}_i$ the instantaneous mass rate of injected propellant and $\bar{\dot{m}}$ the value of $\dot{m}_i$ in steady operation, $\mu_1(t) = [\dot{m}_1(t) - \bar{\dot{m}}]/\bar{\dot{m}}$, with $\dot{m}_1(t)$ the instantaneous mass flow upstream of the capacitance, $\psi(t) = [p_1(t) - \bar{p}_1]/(2\Delta p)$, with $p_1(t)$ the instantaneous pressure at the place in the feeding line where the capacitance representing the elasticity is located, $\bar{p}_1$ the value of p_1 in steady operation and $\Delta p = \bar{p}_1 - \bar{p}$ the injector pressure drop in steady operation, p_0 is the regulated gas pressure for the pressure supply, $P = \bar{p}/(2\Delta p)$, γ is the pressure exponent of the pressure dependence of the combustion process taking place during the time lag, ξ represents the fractional length for the pressure supply, J is the inertia parameter of the line, and E is the elasticity parameter of the line. Guided by [12], we take $u = (p_0 - p_1)/(2\Delta p)$ as a control variable and adopt the following representative numerical values: $\xi = 0.5$, $\gamma = 1$, $P = 1$, $J = 2$, $E = 1$, and $h = 1$. Letting $x(t) = [\varphi(t) \;\; \mu_1(t) \;\; \mu(t) \;\; \psi(t)]^T$, the system (21) reduces to (1) with the delay size h. It is noted that the RHC proposed in [6] cannot be applied to this system because we can not find any stability-guaranteeing terminal weighting matrices using the LMI condition proposed therein. For the simulation study, the weighting matrices Q and R are set to be $Q = 100I$ and $R = I$. The first step for adopting the proposed RHC is to find terminal weighting matrices F_1, F_2, and F_3 guaranteeing the closed-loop stability. Solving the matrix inequality (20) using the algorithm in [13] yields proper F_1, F_2, and F_3. The second step is to obtain W_1 and $W_2(s)$ in (13) for a given T_p by solving (14) and (15). In order to see how the horizon size affects the performance, we design three kinds of RHCs with the horizon sizes, $T_p = 0.2,\ 0.6$, and 1 and apply them to the system with the given initial state $\phi(\theta) = [1 \;\; 1 \;\; 1 \;\; 1]^T$,

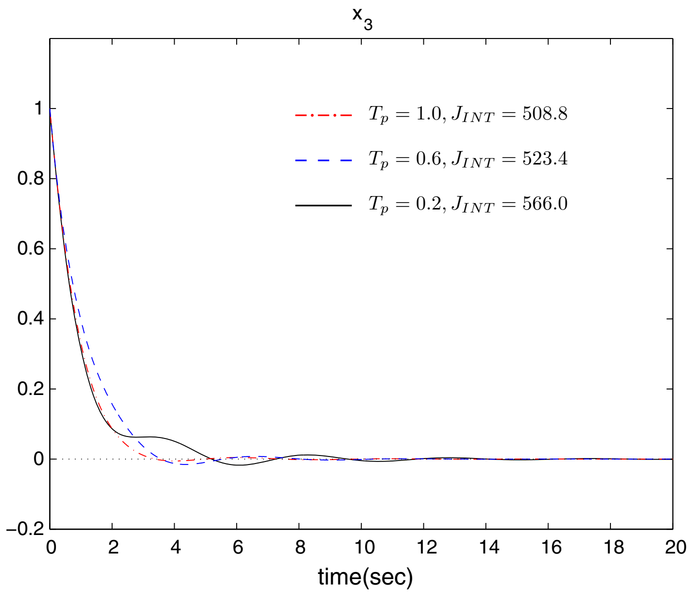

Fig. 1 State trajectories of x_3 due to RHCs with different horizon sizes

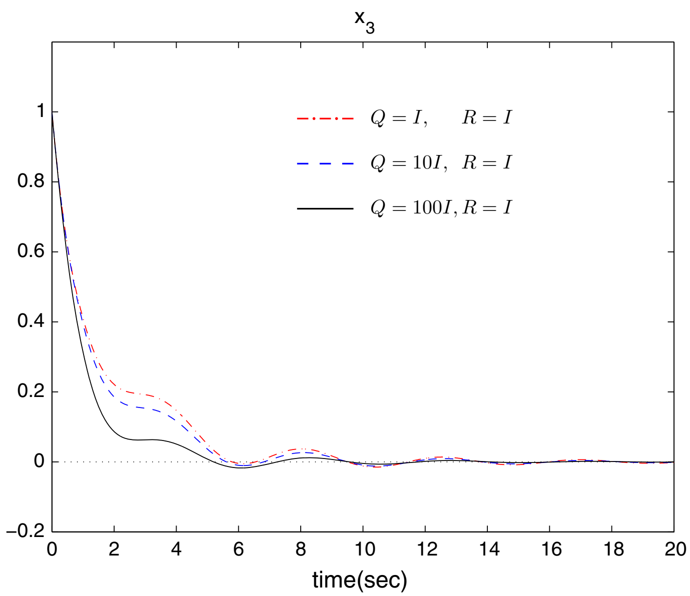

Fig. 2 State trajectories of x_3 due to RHCs with different Q matrices

$-h \le \theta \le 0$. To evaluate the performances of the designed RHCs, we employ an integrated cost given by $J_{INT} = \int_0^{40} [x^T(t)Qx(t) + u^T(t)Ru(t)]dt$. The reason why we chose the integration range from 0 to 40 is that the state trajectories converge to zero before $t = 40$. Figure 1 compares the state trajectories of x_3 generated from those three RHCs. It is observed that three kinds of RHCs are all stabilizing. It is noted that J_{INT} decreases as T_p increases, which implies that the RHC with longer horizon yields better performance. This in turn implies that we can improve the control performance by adjusting the horizon size. However, increasing the horizon size leads to bigger computational burden. This is because we have to perform integration for the increased time range according to (13).

By adjusting the weighting matrices, we can change the state trajectories such that they satisfy the control system requirements. We designed three RHCs for $Q = I$, $Q = 10I$, and $Q = 100I$ to find out how the weighting matrix Q affects the system response. Figure 2 compares the resultant trajectories of three RHCs. It is noted that the faster the trajectory converges to zero the bigger value of Q we use, which is naturally expected.

6 Conclusions

In this paper, a generalized version of the stabilizing RHC was proposed for state-delay systems. We proposed a new type of a receding horizon cost function with three terminal weighting terms and derived the corresponding optimal control. The RHC based on the optimal control is obtained from the solution to nonlinear partial differential matrix equations solved backward in time. An LMI condition was also proposed for stability-guaranteeing terminal weighting matrices. It was shown through simulation that the proposed RHC guarantees closed-loop stability for a wider class of state-delay systems than the existing one since a more generalized cost function is employed.

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