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Author's reply: Comments on delay-dependent robust H_∞ control for uncertain systems with a state-delay[☆]

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Abstract

In the above-mentioned comment, the authors point out a technical problem with the paper [Lee, Y. S., Moon, Y. S., Kwon, W. H., & Park, P. G. (2004). Delay-dependent robust H_∞ control for uncertain systems with a state-delay. *Automatica, 40(1), 65–72*]. We show this technical problem can be solved by changing the proof of Theorem 3.1.
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We would like to thank X. G. Li and X. J. Zhu for their interest in our article (Lee, Moon, Kwon, & Park, 2004). Their comment let us realize that there is a technical problem in our original paper (Lee et al., 2004). In this reply, we will state what the problem is and how it can be solved.

In our commented-on paper (Lee et al., 2004), we use a functional V represented by

$$V = V_1 + V_2 + V_3 + V_4,$$

where

$$V_4 \triangleq \int_0^t \int_{\beta-h}^{\beta} \begin{bmatrix} x(\beta) \\ \dot{x}(\beta) \\ \dot{x}(\alpha) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X_{11} & X_{12} & Y_1 \\ \star & X_{22} & Y_2 \\ \star & \star & Z \end{bmatrix} \begin{bmatrix} x(\beta) \\ \dot{x}(\beta) \\ \dot{x}(\alpha) \end{bmatrix} \mathrm{d}\alpha\, \mathrm{d}\beta$$

and

$$\begin{bmatrix} X_{11} & X_{12} & Y_1 \\ \star & X_{22} & Y_2 \\ \star & \star & Z \end{bmatrix} \geqslant 0, \quad Z > 0. \tag{1}$$

Refer to the paper (Lee et al., 2004) for V_1, V_2, and V_3. In the above-mentioned comment paper (Li & Zhu, 2006), the authors raised two arguments, summarized as follows:

(i) V is not a Lyapunov–Krasovskii functional because V does not satisfy the condition: $V(x_t) = 0$ when $x_t = 0$.
(ii) Therefore the Lyapunov–Krasovskii theorem cannot be applied. This, in turn, implies that Theorem 3.1 may be wrong. Since Theorem 3.1 affects the remaining results, the whole results in the paper may be wrong.

In what follows, we give our reply to the above arguments.

About (i): We agree with the authors that V is not a Lyapunov–Krasovskii functional. Therefore, we will choose a Lyapunov–Krasovskii functional differently from the one in the original paper such that it consists of V_1, V_2, and V_3 only.

About (ii): Theorem 3.1 still holds true. However, we should modify the proof slightly for completeness. In the modified proof, the notation V_4 is not used. Instead, the notation $\zeta(x_t)$, which corresponds to $\dot{V}_4$ in the original paper, is used.

The proof of Theorem 3.1 can be complete if we change five parts in the original paper as follows:

- The first three lines on the second column of p. 67 should be replaced by "Let us choose a Lyapunov–Krasovskii

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functional candidate $V(x_t)$

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t)\text{''}.$$

- The eighth to ninth lines on the second column of p. 67, which show the definition of $V_4(x_t)$, should be deleted.
- The 12th line on the second column of p. 67 should be replaced by

$$\text{``}\ P_1 > 0, \quad Z > 0, \quad Q > 0.\text{''}$$

- The first to third lines on the first column of p. 68, which show the representation of $\dot{V}_4$, should be deleted.
- The 10th to 12th lines on the first column of p. 68 should be replaced by "Let's define $\zeta(x_t)$ as follows:

$$\zeta(x_t) \triangleq \int_{t-h}^{t} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \dot{x}(\alpha) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X_{11} & X_{12} & Y_1 \\ \star & X_{22} & Y_2 \\ \star & \star & Z \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \dot{x}(\alpha) \end{bmatrix} \mathrm{d}\alpha,$$

where

$$\begin{bmatrix} X_{11} & X_{12} & Y_1 \\ \star & X_{22} & Y_2 \\ \star & \star & Z \end{bmatrix} \geqslant 0.$$

It is apparent that $\zeta(x_t) \geqslant 0$. Using the relation $h \leqslant \bar{h}$ and $\zeta(x_t) \geqslant 0$, the upper bound on J_{zw} is written as follows:

$$\begin{aligned} J_{zw} &\leqslant \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(x_t)]\,\mathrm{d}t \\ &\leqslant \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(x_t) + \zeta(x_t)]\,\mathrm{d}t\text{''}. \end{aligned}$$

Remark 1. In our original paper, we argued that a new Lyapunov–Krasovskii functional is proposed. However, this argument should be withdrawn. Results similar to Theorem 3.1 appear in Fridman and Shaked (2002) and Gao and Wang (2003). It is mentioned that the novelty of our original paper is the new method of derivation.

Remark 2. It seems that adding a positive term ζ in order to bound J_{zw} may cause additional conservatism in L_2 gain. It may be true. However, the conservatism can be made arbitrarily small by adjusting X_{11}, X_{12}, X_{22}, Y_1, Y_2, and Z. Furthermore, those matrix variables actually help the bounded real lemma be formulated in terms of LMIs (6) and (7) given in our original paper.

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